# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS



REPORT No. 216

# THE REDUCTION OF AIRPLANE FLIGHT TEST DATA TO STANDARD ATMOSPHERE CONDITIONS

By WALTER S. DIEHL and E. P. LESLEY



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# AERONAUTICAL SYMBOLS.

## 1. FUNDAMENTAL AND DERIVED UNITS.

| | Symbol. | Metric. | | English. | |
|---|---|---|---|---|---|
| | | Unit. | Symbol. | Unit. | Symbol. |
| Length | $l$ | meter | m. | foot (or mile) | ft. (or mi.). |
| Time | $t$ | second | sec. | second (or hour) | sec. (or hr.). |
| Force | $F$ | weight of one kilogram | kg. | weight of one pound | lb. |
| Power | $P$ | kg.m/sec | | horsepower | HP |
| Speed | | m/sec | m. p. s. | mi/hr | M. P. H. |

## 2. GENERAL SYMBOLS, ETC.

Weight, $W=mg$.

Standard acceleration of gravity, $g=9.806\text{m/sec.}^2=32.172\text{ft/sec.}^2$

Mass, $m=\frac{W}{g}$

Density (mass per unit volume), $\rho$

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

Specific weight of "standard" air, 1.223 kg/m.$^3$ = 0.07635 lb/ft.$^3$

Moment of inertia, $mk^2$ (indicate axis of the radius of gyration, $k$, by proper subscript).

Area, $S$; wing area, $S_w$, etc.

Gap, $G$

Span, $b$; chord length, $c$.

Aspect ratio $=b/c$

Distance from $c.\,g.$ to elevator hinge, $f$.

Coefficient of viscosity, $\mu$.

## 3. AERODYNAMICAL SYMBOLS.

True airspeed, $V$

Dynamic (or impact) pressure, $q=\frac{1}{2}\rho V^2$

Lift, $L$; absolute coefficient $C_L=\frac{L}{qS}$

Drag, $D$; absolute coefficient $C_D=\frac{D}{qS}$

Cross-wind force, $C$; absolute coefficient $C_c=\frac{C}{qS}$.

Resultant force, $R$

(Note that these coefficients are twice as large as the old coefficients $L_c$, $D_c$.)

Angle of setting of wings (relative to thrust line), $i_w$

Angle of stabilizer setting with reference to thrust line $i_t$

Dihedral angle, $\gamma$

Reynolds Number $=\rho\frac{Vl}{\mu}$, where $l$ is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi/hr., normal pressure, 0°C: 255,000 and at 15.6°C, 230,000;

or for a model of 10 cm. chord, 40 m/sec., corresponding numbers are 299,000 and 270,000.

Center of pressure coefficient (ratio of distance of $C.P.$ from leading edge to chord length), $C_p$.

Angle of stabilizer setting with reference to lower wing. $(i_t - i_w)=\beta$

Angle of attack, $\alpha$

Angle of downwash, $\epsilon$

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## THE REDUCTION OF AIRPLANE FLIGHT TEST DATA TO STANDARD ATMOSPHERE CONDITIONS

By Walter S. Diehl and E. P. Lesley

### SUMMARY

This paper was prepared for the National Advisory Committee for Aeronautics in order to supply the need of practical methods of reducing observed performance to standard conditions with a minimum of labor. The first part gives a very simple approximate method of reducing performance in climb, and is particularly adapted to work not requiring extreme accuracy. The second part gives a somewhat more elaborate and more accurate method which is well suited to general flight test reduction. The third part gives the conventional method of calibrating airspeed indicators and reducing the indicated speeds to true airspeeds. An appendix gives working tables and charts for the standard atmosphere.

## PART I

## THE REDUCTION OF TEST DATA IN CLIMB TO STANDARD CONDITIONS—A SIMPLE APPROXIMATE METHOD

By Walter S. Diehl

### SUMMARY

This paper was prepared for the National Advisory Committee for Aeronautics to illustrate a simplified method of reducing observed airplane performance data to standard conditions. The method is based on the assumption that under any normal conditions of pressure and temperature, that is, at any given air density, the instantaneous climb (or speed) has the same value that it would have at the same density in standard atmosphere. This assumption allows the corrections to be made to the altitude rather than to a rate of climb deduced from successive altimeter or aneroid readings. As a result, the calculations are reduced to approximately 25 per cent of the amount required by the old method. The results by both methods are in substantial agreement.

The principle of correcting to altitude in the standard atmosphere is also applied to the reduction of airspeed meter readings to true airspeeds.

### INTRODUCTION

The performance of an airplane must depend on the relations between the power required and the power available. Consequently, in a study of the variations of performance due to changes in atmospheric conditions we must consider the variations of each factor. For any given airplane, at a given air speed, the power required will vary as the square root of the air density. The power available from a conventional airplane engine varies with both pressure and temperature, according to the relations given in National Advisory Committee for Aeronautics Technical Report No. 171. The propeller efficiency at a given slip is probably independent of the pressure and slightly dependent on temperature (i. e., in so far as the viscosity is concerned). The resultant effect of the variations is very difficult to predict unless we assume

a standard atmosphere in which there are definite relations between pressure, temperature, density, and altitude. For the same reasons the observed performance of a given airplane is variable from day to day, so that it is necessary to reduce the observations to standard conditions in order to obtain comparable values.

Unfortunately the reduction of performance data to standard conditions is a tedious process when the usual methods are employed; so tedious, in fact, that few engineers have had the patience to master the methods, and so uncertain that the final results by different methods are not always in accordance with each other. It is the purpose of this paper to present a very simple method based on the assumptions that the rate of climb decreases uniformly with increase in altitude and that the instantaneous climb for normal pressures and temperatures; that is, for any given density, has the same value that it would have at the same density in standard atmosphere. The former assumption is verified by test data and is frequently taken as an accepted fact. The second assumption not only seems to be justified in view of the very small average value of the corrections which must be made for strict accuracy, but it also leads to a great simplification in the method of reduction to standard conditions.

In Part III of this report the conventional method for reducing airspeed meter readings to true airspeeds has been developed along the same lines followed in reducing performance in climb. When these methods are used it will be found that the data from a complete performance test may be quickly reduced to standard conditions. As will be shown later, the results are in substantial agreement with those obtained by the use of the more complicated methods.

For the information of the reader who wishes to compare the new method with the conventional methods reference is made to Chapter IX of Bairstow's Applied Aerodynamics, or to the various reports of the British Advisory Committee for Aeronautics, R. & M. 324, 474, 608, etc.

## OUTLINE OF THE METHOD

The following brief outline will indicate the steps taken to reduce an observed performance in climb to standard conditions: First, obtain the atmospheric pressures $p$, either from aneroid readings or from the altimeter readings $Z_i$. Next there is found the altitude in standard atmosphere $Z_e$, at which the density is the same as that determined by each pressure $p$, and the corresponding observed air temperature $t$. A time-of-climb curve is then plotted with the equivalent altitudes $Z_e$ as ordinates and the corresponding times $T$ as abscissas. The rates of climb are determined at any desired number of points along this curve by the corresponding tangents. The values so found are plotted as abscissas to some convenient scale against $Z_e$ as ordinates and a straight line is drawn through the points which will give the most probable climb at each altitude. In most cases this line will be well defined. From it the corrected time-of-climb curve is calculated. A table may now be made giving, according to requirements, a series of altitudes in standard atmosphere with the corresponding rate of climb and time of climb. This completes the reduction of the climb data to standard conditions.

The data required for this method of reduction are: Either aneroid pressure readings or the initial pressure $p_o$ (i. e., the barometric pressure at which the altimeter reads zero), and the altimeter readings $Z_i$, time of climb $T$, and corresponding air temperatures $t$. Obviously such data as weight of airplane, type and condition of engines, characteristics of propeller, weather conditions, etc., should be taken for general information. R. P. M. and airspeed meter readings are also desirable and should be taken along with the test data.

Whenever possible an aneroid which reads pressure directly should be used. This will eliminate the conversion of altimeter readings to pressures and thus give somewhat greater accuracy to the reduction.

## CONVERSION OF ALTIMETER READINGS TO PRESSURES

Standard altimeters in use in the United States are constructed and graduated to read altitudes corresponding to pressures given by the modified Halley's equation

$$Z_i = 62{,}900 \log_{10}\left(\frac{29.90}{p}\right) \qquad (1)$$

where $Z_i$ is the altimeter reading in feet and $p$ the barometric pressure in inches. The constant 62,900 in this equation corresponds to a mean air temperature of +10° C. and to the average humidity. Only when these conditions are fulfilled does an altimeter read the true altitude. However, the instrument is actuated by pressure, and owing to its uniformly divided scale the barometric pressure corresponding to any reading may readily be obtained from equation (1) written in the form

$$Z_i = 62{,}900 \log_{10}\left(\frac{p_o}{p}\right) \tag{2}$$

where $p_o$ is the barometric pressure at which the instrument reads zero. $p$ may be calculated from (2) or read directly from the curve in Figure 1, which is a plot of the data in Table I.

Specimen calculations of $p$ will be found in columns 4 and 5 of Table II, using actual test data taken from Table 10, Chapter IX, of Applied Aerodynamics (Bairstow). The value of $\frac{p}{p_o}$ corresponding to each altimeter reading has been read from the curve in Figure 1. The next step is to find the altitude in standard atmosphere having the density determined by $p$ and $t$. This altitude will be called the "equivalent altitude" and denoted by the symbol $Z_e$.

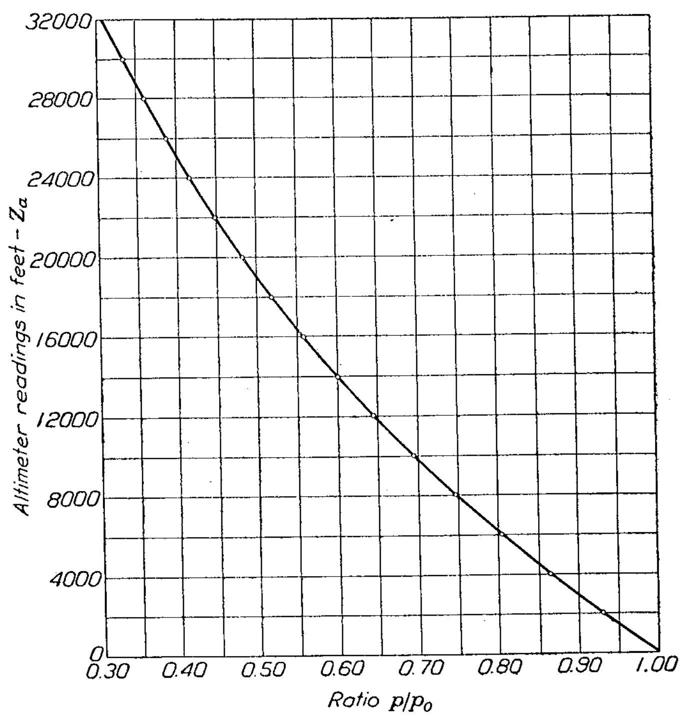



FIG. 1.—Relation between pressure ratio and altimeter reading. (Altimeter reading zero when $p = p_o$)

## EQUIVALENT ALTITUDE IN STANDARD ATMOSPHERE

In the standard atmosphere there are definite relations between $p$, $t$, $\rho$, and $Z$, but we may, without appreciable error, neglect the deviations from normal in $p$ and $t$ and say that the altitude in standard atmosphere corresponding to the density determined by actual values of $p$ and $t$ is an "equivalent altitude" for the observed conditions. That is, the equivalent altitude in standard atmosphere $Z_e$ for any given $p$ and $t$ is that altitude in the standard atmosphere at which the density is that determined by the given values of $p$ and $t$.

$Z_e$ may be calculated from the equations in National Advisory Committee for Aeronautics Technical Note No. 99, or read directly from the chart in Figure 2. The values in column 6 of Table II were read from Figure 2.

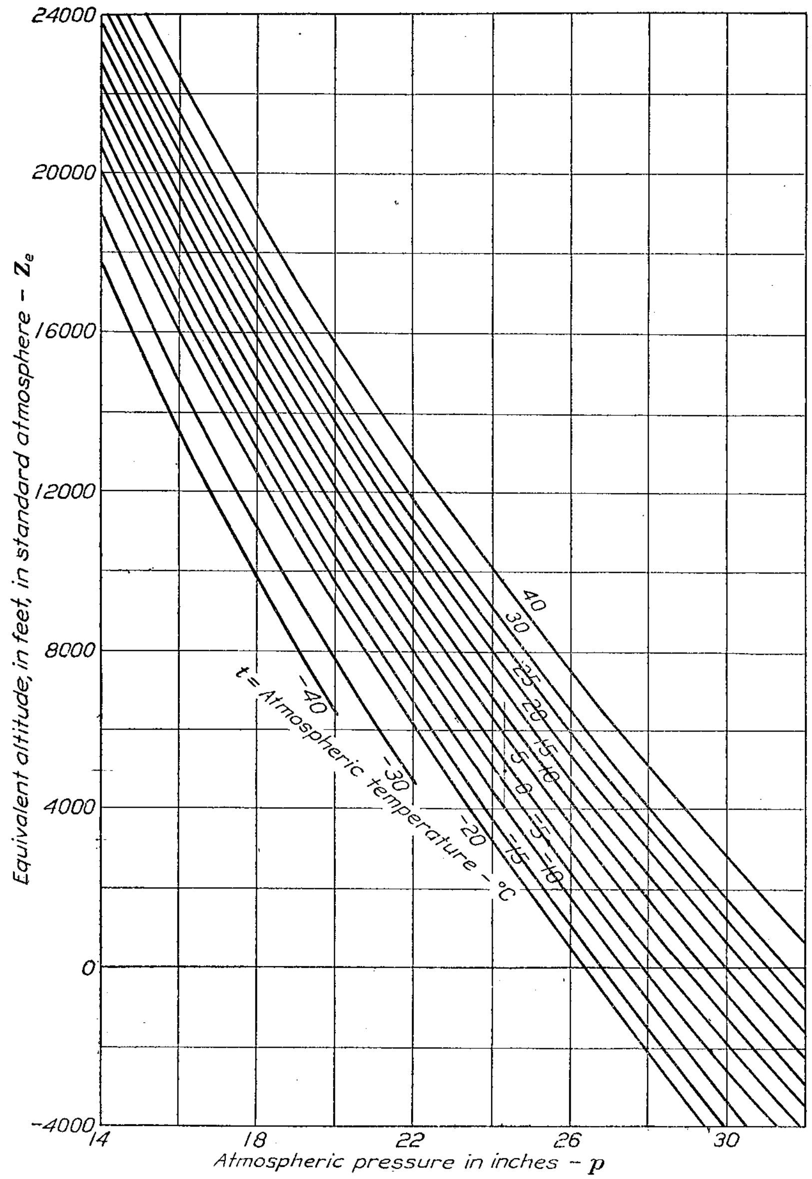



FIG. 2.—Equivalent altitude in standard atmosphere for any given pressure and temperature for use in reducing observed performance to standard conditions

## RATE OF CLIMB IN STANDARD ATMOSPHERE

A plot is now made of $Z_e$ against the recorded time $T$, as in Figure 3, which employs the data from Table II. It is desirable to adopt the largest convenient scale for both variables in this plot, since an open scale enables the slope of the tangent which determines the rate of climb to be read with greater accuracy. When drawing in the curve of $Z_e$ against $T$ great care must be taken in order that it may truly represent the observed data. Some judgment is required to draw it correctly. The curve should pass through every point if practicable, although points which are obviously high or low may be ignored.

The rate of climb at any desired altitude is obtained directly from the slope of tangent at that altitude. The tangent may be obtained in several ways, but the most satisfactory method seems to be the old one in which two transparent triangles are used, one being held fixed and the other slid along it until the parallel to the tangent passes through the origin. This line then intersects the 10-minute abscissa at an altitude which is obviously 10 times the desired rate of climb.

Fortunately, the errors, which are cumulative in time readings, do not affect the rate of climb obtained from the slope of any portion of the climb curve, which is a "true climb." For example, it frequently happens that a cautious pilot does not obtain the maximum climb until an altitude of several thousand feet is reached, or an inexperienced pilot may not obtain the best climb at high altitudes. In either case, if the best climb be obtained and held for a few thousand feet, it will be sufficient to determine the straight line which represents the best rate of climb plotted against altitude. Consequently, the rates of climb as determined by the slopes of the tangents are plotted against altitude and a straight line drawn through the values which give the most probable climb. Scattered high or low values in rate of climb may be neglected entirely, although the rate of climb is more often low than high when incorrect.

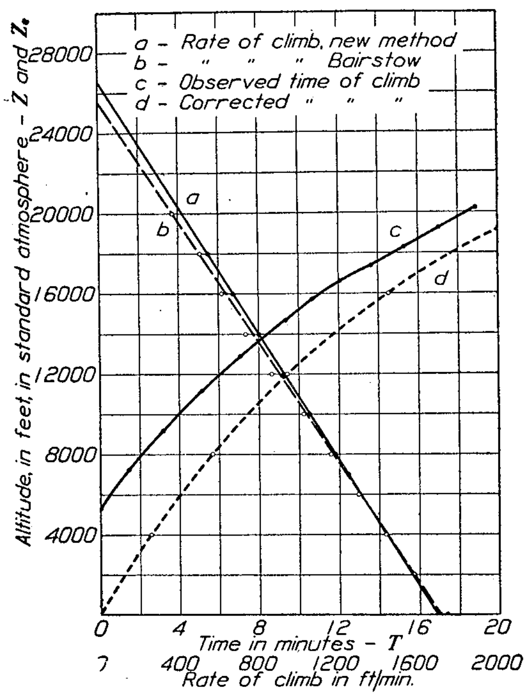



FIG. 3.—Reduction of test data in climb to standard conditions with comparison of results by old and new methods

NOTE.—Observed time of climb is from test data given in Table 10, Chapter IX, "Applied Aerodynamics."

The correct time-of-climb curve may be constructed from the rate-of-climb line either by the use of average rate of climb over short intervals of altitude, or better, by the use of the equation

$$T \text{ minutes} = 2.304 \left(\frac{Z_a}{C_o}\right) \log_{10} \left(\frac{Z_a}{Z_a - Z}\right) \tag{3}$$

where $Z_a$ is the absolute ceiling, $Z$ the altitude to which the time of climb $T$ is desired, and $C_o$ the initial rate of climb. Note that $C_o$ and $Z_a$ are given by the intersections of the rate-of-climb line with the rate-of-climb axis and the altitude axis, respectively.

## COMPARISON OF METHODS

The test data used in Table II to illustrate the simplified method of reduction was taken from Chapter IX, Applied Aerodynamics, where it is reduced to standard conditions according to the British method. Unfortunately the ground pressure is not given with this data and must be assumed. The altitudes are given by Bairstow as "aneroid height" without qualification. It is therefore probable that the instrument had a fixed instead of an adjustable scale. Assuming this to be the case, the initial pressure must be $p_o = 29.92$ inches, the value here adopted.

When these data are reduced by the simplified method, the time-of-climb curve plots as in Figure 3. The tangents to this curve give the rates of climb indicated by dots. Bairstow's values for rate of climb are indicated by circles. Assuming that the rate of climb is a linear function of altitude, a straight line has been drawn through each set of reduced rates of climb. It will be noted that the difference is comparatively small and that it is greatest at high altitudes when the corrections are most difficult to apply. The difference in the indicated absolute ceiling is 1,000 feet in 26,000 feet, about 4 per cent. This is ordinarily less than the precision of the test data.

In order that the differences between the two methods may be made more clear, Table III has been prepared. In this table the rate of climb and time of climb are given at a series of altitudes for three cases: (1) The actual figures obtained from reduction of the test data by the old method; (2) the same figures corrected on the assumption that the rate of climb is linear with altitude; and (3) the corresponding results obtained by the simplified method of reduction.

TABLE I

RELATION BETWEEN ALTIMETER READING AND ATMOSPHERIC PRESSURE

$$Z_a = 62{,}900 \log_{10}\left(\frac{p_o}{p}\right)$$

| $Z_a$ | $\frac{p}{p_o}$ |
|---|---|
| *Feet* | |
| 0 | 1.0000 |
| 1,000 | .9640 |
| 2,000 | .9294 |
| 3,000 | .8954 |
| 4,000 | .8638 |
| 5,000 | .8327 |
| 6,000 | .8028 |
| 7,000 | .7735 |
| 8,000 | .7461 |
| 9,000 | .7193 |
| 10,000 | .6935 |
| 12,000 | .6445 |
| 14,000 | .5990 |
| 16,000 | .5567 |
| 18,000 | .5174 |
| 20,000 | .4809 |
| 22,000 | .4469 |
| 24,000 | .4154 |
| 26,000 | .3861 |
| 28,000 | .3588 |
| 30,000 | .3335 |

TABLE II

REDUCTION OF PERFORMANCE TEST DATA ON HIGH-SPEED SCOUT

[Test data. $p_o$=29.92]

| $T$ | $Z_a$ | $\frac{t}{oC}$ | $\frac{p}{p_o}$ | $p$ | $Z_e$ |
|---|---|---|---|---|---|
| *Min., sec.* | *Altimeter feet* | | | *Inches* | *Feet* |
| -------- | 0 | 27 | 1.000 | 29.92 | +1,500 |
| 0 | 4,000 | 18 | .864 | 25.85 | 5,300 |
| 1.28 | 6,000 | 15 | .803 | 24.05 | 7,250 |
| 3.12 | 8,000 | 11 | .746 | 22.32 | 9,150 |
| 5.07 | 10,000 | 7 | .693 | 20.72 | 11,150 |
| 7.04 | 12,000 | 3 | .644 | 19.27 | 12,900 |
| 9.22 | 14,000 | −1 | .600 | 17.95 | 14,700 |
| 10.41 | 15,000 | −2 | .518 | 17.30 | 15,750 |
| 12.03 | 16,000 | −4 | .557 | 16.68 | 16,650 |
| 13.38 | 17,000 | −6 | .538 | 16.10 | 17,400 |
| 15.18 | 18,000 | −8 | .518 | 15.50 | 18,300 |
| 17.04 | 19,000 | −10 | .499 | 14.93 | 19,200 |
| 18.50 | 20,000 | −10 | .481 | 14.41 | 20,250 |

Test data in columns 1-3 are taken from Table X, Chapter IX of Applied Aerodynamics, Bairstow.

TABLE III

COMPARISON OF PERFORMANCE REDUCED TO STANDARD CONDITIONS BY OLD AND NEW METHODS

| $Z$ | Bairstow [1] | | Bairstow [2] | | New method | |
|---|---|---|---|---|---|---|
| | $\frac{dZ}{dt}$ | $T$ | $\frac{dZ}{dt}$ | $T$ | $\frac{dZ}{dt}$ | $T$ |
| *Feet* | *Feet per minute* | *Minutes* | *Feet per minute* | *Minutes* | *Feet per minute* | *Minutes* |
| 0 | 1,740 | 0 | 1,705 | 0 | 1,690 | 0 |
| 2,000 | 1,570 | 1.21 | 1,510 | 1.21 | 1,565 | 1.21 |
| 4,000 | 1,430 | 2.54 | 1,435 | 2.55 | 1,435 | 2.58 |
| 6,000 | 1,295 | 4.02 | 1,305 | 4.00 | 1,310 | 4.00 |
| 8,000 | 1,160 | 5.75 | 1,170 | 5.63 | 1,180 | 5.63 |
| 10,000 | 1,020 | 7.49 | 1,035 | 7.43 | 1,050 | 7.40 |
| 12,000 | 865 | 9.63 | 905 | 9.52 | 925 | 9.43 |
| 14,000 | 735 | 12.15 | 770 | 11.90 | 800 | 11.70 |
| 16,000 | 615 | 15.12 | 640 | 14.72 | 675 | 14.50 |
| 18,000 | 505 | 18.70 | 505 | 18.23 | 550 | 17.60 |
| 20,000 | 370 | 23.30 | 370 | 22.80 | 420 | 21.90 |

[1] Table 18, Chapter IX, Applied Aerodynamics. (Actual reduced values.)

[2] Same data, assuming $\left(\frac{dZ}{dt}\right)$ linear with $Z$.

## PART II

# REDUCTION OF AIRPLANE PERFORMANCE IN CLIMB TO STANDARD CONDITIONS

By E. P. Lesley

### SUMMARY

This is a description of the method proposed and used by the staff of the Langley Memorial Aeronautical Laboratory for the reduction of data secured in flight tests to the conditions of the standard atmosphere. It is assumed that, for the moderate changes of pressure and temperature generally encountered in passing from conditions of the actual atmosphere to those of the standard, the power and R. P. M. of the engine, as well as the thrust of the propeller and the lift and drag of the airplane, depend only upon the density, or upon the specific weight, of the air. Under this assumption, a simple method for transforming observed or recorded altitudes and times to altitudes and times for the standard atmosphere is described and illustrated. Rates of climb are determined by drawing tangents to the time-altitude curve.

## INTRODUCTION

Experience with several methods of reducing flight-test data to the conditions of the standard atmosphere has led to the formulation and adoption of the following, which, while in some respects not new, and in others possibly not as accurate as might be desired, yet offers the advantages of simplicity, the reduction of the original barograph to the time-altitude curve for the standard atmosphere without preliminary plotting of rates of climb or other curves, and sufficient accuracy for any purpose such reduced data serve.

While the evidence of tests indicates that engine performance varies with air pressure and temperature, even though density remains constant, the introduction of corrections for such variation complicates the problem of reduction very considerably; and since these corrections are usually small, generally within the probable error in the original data, and not determinate without preliminary test of the engine, it appears that they may be neglected without serious consequences. Therefore it is assumed that the power and R. P. M. of the engine are constant for constant density, and do not change with temperature and pressure in passing from the encountered to the standard atmosphere, provided only that the density remains the same. This assumption being granted, it follows that airspeed and rate of climb are constant for constant density.

## STANDARD ATMOSPHERE

The air of the standard atmosphere is considered dry. The pressure at zero altitude is 760 mm. of mercury and the temperature is 15° centigrade. The specific weight is 1.2255 kg./m.$^3$ The temperature gradient is $-6.5°$ centigrade per thousand meters to an altitude of 10,769 meters, at which a temperature of $-55°$ centigrade is reached. From this point upwards the temperature is assumed constant. Boyle's law, that density, or specific weight, varies directly as the absolute pressure and inversely as the absolute temperature, is assumed to apply throughout.

Using metric notation, in which

$Z$ = altitude, meters,
$t$ = temperature, degrees centigrade,
$T_m$ = harmonic mean temperature, degrees centigrade, absolute,
$p$ = pressure, millimeters of mercury,
$\rho$ = density, mass per unit volume, kg.–m.–sec.,
$g\rho$ = specific weight, kg./m.,$^3$

the conditions of the standard atmosphere may be formulated as follows:

For altitudes up to 10,769 meters

$$t = 15 - 0.0065Z \qquad 1$$

$$g\rho = 0.4644\frac{p}{t+273} \qquad 2$$

$$p = \left(\frac{44,308 - Z}{12,540}\right)^{5.256} \qquad 3$$

For altitudes above 10,769 meters

$$t = -55° \qquad 4$$

$$g\rho = 0.0021303\ p \qquad 5$$

$$\mathrm{Log}_{10}p = 2.880814 - \frac{Z}{67.4072\,T_m} \qquad 6$$

For all altitudes

$$dZ = \frac{-13.59\ dp}{g\rho} \qquad 7$$

Using English notation, in which

$Z$ = altitude, feet,
$t$ = temperature, degrees Fahrenheit,
$T_m$ = harmonic mean temperature, degrees Fahrenheit, absolute
$p$ = pressure, inches of mercury,
$\rho$ = density, mass per unit volume, lb.–ft.–sec.,
$g\rho$ = specific weight, lb./cu. ft.,

For altitude up to 35,332 feet

$$t = 59 - 0.003566Z \tag{8}$$

$$g\rho = \frac{1.3256p}{t + 459.4} \tag{9}$$

$$p = \left(\frac{145{,}365 - Z}{76{,}140}\right)^{5.256} \tag{10}$$

For altitudes above 35,332 feet

$$t = -67 \tag{11}$$

$$g\rho = 0.003378p \tag{12}$$

$$\log_{10}p = 1.475976 - \frac{Z}{122.862\,T_m} \tag{13}$$

For all altitudes

$$dZ = \frac{-70.67\ dp}{g\rho} \tag{14}$$

From formulas 1 to 14 the temperature, pressure, and specific weight or density at any altitude in the standard atmosphere are readily calculated. In the appendix, Tables VI and VII show the above quantities for metric and English units, respectively, while the associated Figures 7 and 8 show the specific weight altitude relations as tabulated.

The altitude in the standard atmosphere for specific weight equal to that of the air encountered in flight may be either read directly from the charts or interpolated from the tables, the specific weight of the encountered air having been first computed by formula (2) or (9). The charts are found to be more convenient but the tables more accurate. The former are, however, sufficiently accurate in the usual case.

## REDUCTION OF FLIGHT DATA TO STANDARD CONDITIONS

The observed or recorded data consist essentially of time, air pressure, and air temperature. Indicated airspeed, revolutions per minute, various engine pressures and temperatures, angle of attack, and any other desired data may be added to these. The pressure is often recorded in terms of indicated altitude, but this is, by a calibration of the instrument, readily converted into pressure in millimeters or inches of mercury.

The essential observed data, together with computations for the time-altitude relation for the standard atmosphere, are conveniently arranged under column headings as follows:

| COLUMN NO. | SYMBOL | QUANTITY |
|---|---|---|
| 1 | $T_o$ | Time, observed. |
| 2 | $\Delta T_o$ | Increment of time, observed. |
| 3 | $p_o$ | Pressure, observed. |
| 4 | $\Delta p_o$ | Increment of pressure, observed. |
| 5 | $t_o$ | Temperature, observed. |
| 6 | $g\rho$ | Specific weight (computed from observed temperature and pressure). |
| 7 | $g\rho_m$ | Mean specific weight for altitude increment. |
| 8 | $\Delta Z_a$ | Increment of altitude, absolute. |
| 9 | $Z_s$ | Altitude in standard atmosphere for equal specific weight. |
| 10 | $\Delta Z_s$ | Increment of altitude, standard atmosphere. |
| 11 | $\Delta T_s$ | Increment of time, standard atmosphere. |
| 12 | $T_s$ | Time, standard atmosphere. |

In the above it is to be noted that the observed times, pressures, and temperatures are presumed to have the correct values, or values determined from calibration curves of instruments used.

$\Delta T_o$ is the interval of time between two consecutive observations and equals

$$T_{o_2} - T_{o_1},\ T_{o_3} - T_{o_2},\ T_{o_4} - T_{o_3},\ \text{etc.}$$

In like manner $\Delta p_o$ is equal to $p_{o_2} - p_{o_1}$, $p_{o_3} - p_{o_2}$, $p_{o_4} - p_{o_3}$, etc., being consequently negative if observed pressures are decreasing.

$g\rho$ is determined from the equation

$$g\rho = \frac{0.4644p}{t+273}$$

if metric units are used, or from

$$g\rho = \frac{1.3256p}{t+459}$$

if the English system is employed,

$$g\rho_m \text{ is equal to } \frac{g\rho_1 + g\rho_2}{2},\ \frac{g\rho_2 + g\rho_3}{2},\ \frac{g\rho_3 + g\rho_4}{2},\ \text{etc.}$$

$\Delta Z_a$, the increment of absolute altitude, is determined from equation (7) or (14), it being assumed that the relation as expressed for the differentials holds true for the finite increments $\Delta Z$ and $\Delta p$, and that the change in specific weight from $g\rho_1$ to $g\rho_2$, $g\rho_2$ to $g\rho_3$, etc., is rectilinear. Consequently

$$\Delta Z_a = \frac{-13.59\Delta p_o}{g\rho_m} \text{ for metric units,}$$

$$\Delta Z_a = \frac{-70.67\Delta p_o}{g\rho_m} \text{ for English units.}$$

$\Delta Z_a$ is thus positive for decreasing values of $p_o$, since $\Delta p$ in such case is negative.

$Z_s$ is read directly from Figure 7 or Figure 8, or is interpolated from Table VI or Table VII for the various values of $g\rho$,

$$\Delta Z_s = Z_{s_2} - Z_{s_1},\ Z_{s_3} - Z_{s_2},\ \text{etc.}$$

$\Delta T_s$ is deduced from $\Delta T_o$ through the relation

$$\frac{\Delta Z_a}{\Delta T_o} = \frac{\Delta Z_s}{\Delta T_s},$$

which arises from the assumption that the engine power is constant for constant density, and that therefore rates of climb are the same for the encountered atmosphere as for the standard atmosphere of equal density. Therefore

$$\Delta T_s = \Delta T_o \frac{\Delta Z_s}{\Delta Z_a}.$$

$T_s$ is the summation of $\Delta T_s$.

Table IV shows the reduction of data from an actual flight test by the above method. As may be noted, the altitude in the standard atmosphere at the start is not zero. This will generally be the case, for although the flight test may be started at near to sea level, it will be the very rare exception that the density encountered at the start will be that of zero altitude in the standard atmosphere.

If it is desired that the time-altitude curve for the standard atmosphere pass through the origin, an initial time increment $\Delta T_{s_0}$, must be included at the beginning of column $\Delta T_s$.

$$\Delta T_{s_0} = \Delta T_{s_1} \frac{Z_{s_1}}{\Delta Z_{s_1}}.$$

$\Delta T_s$ is thus positive if $Z_{s_1}$ is positive, and negative if $Z_{s_1}$ is negative.

It sometimes arises that the altitudes of the standard atmosphere for two or more of the observations near the start are negative. In such case it is found most convenient to neglect all but the last of such observations, making the starting point at that observation nearest to zero altitude standard atmosphere. Table IV has the time increment, $\Delta T_{s_0}$, included at the top of column for $\Delta T_s$. The final reduced data, altitude in standard atmosphere $Z_s$, and time for standard atmosphere $T_s$, are plotted in Figure 4. The resulting rate of climb curve, determined by taking tangents of the altitude-time curve, is shown.

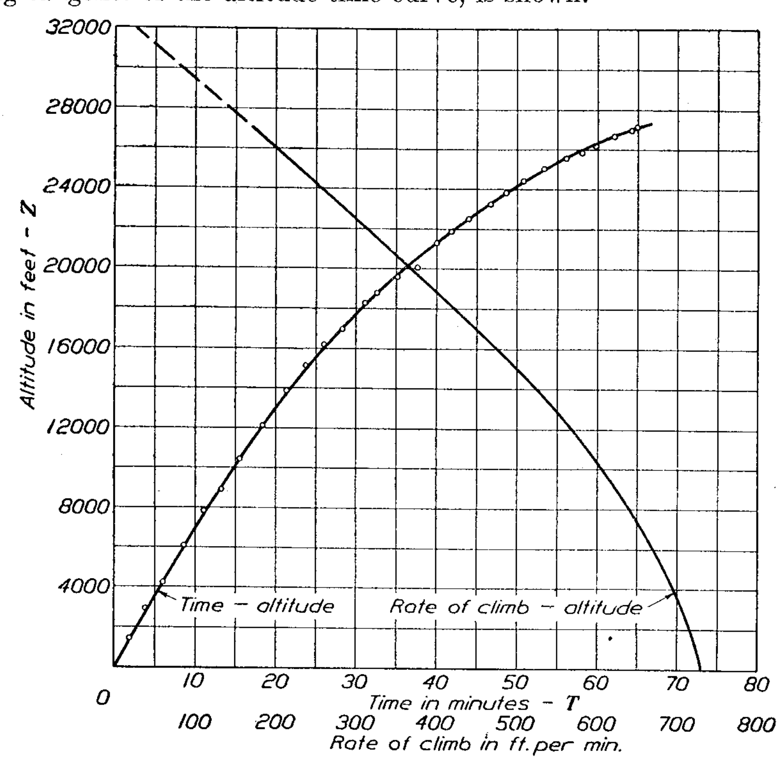



FIG. 4.—DH 4B airplane. Roots supercharger. Flight 18 B standard atmosphere. Reduction of data by Langley Memorial Aeronautical Laboratory Method

## DISCUSSION

It may be noted that in the example given the points for time altitude in the standard atmosphere do not all lie upon a smooth, fair curve. The dispersion from such a curve in however, not greater than would be prevalent for points of absolute altitude and observed time, since the slope between corresponding consecutive points is, from the method of reduction, the same for the two cases. The result of this method is that the time-altitude curve for the standard atmosphere has, at altitudes of equal density and if corresponding points are gives, equal value, the same slope as the curve for observed time versus absolute altitude. Rates of climb in the standard atmosphere are thus the same as rates of climb in the encountered atmosphere of equal density.

It does not generally seem possible to draw through all of the points for altitude and time, whether these be absolute altitude and observed time or altitude in the standard atmosphere and time for the standard atmosphere, what may be properly called a smooth curve, particularly if the observations are made at short time intervals. This is true even if the test flight has been conducted with the best endeavor of the pilot to secure a smooth barograph. If, however, the observations are at comparatively long intervals, a smooth, fair curve is more readily drawn through all points. In the example given the data were recorded at 17-second intervals by means of photograph apparatus. The pressures and temperatures given in Table IV were taken at 2½-minute intervals from the carefully plotted but irregular line drawn through all observations.

TABLE IV

COMPUTATIONS FOR REDUCTION TO THE STANDARD ATMOSPHERE
DH-4B ROOTS SUPERCHARGER

Flight 18 B, September 27, 1923

| 1<br>$T_0$ | 2<br>$\Delta T_0$ | 3<br>$p_0$ | 4<br>$\Delta p_0$ | 5<br>$t_0$ | 6<br>$g\rho$ | 7<br>$g\rho_m$ | 8<br>$\Delta Z_n$ | 9<br>$Z_s$ | 10<br>$\Delta Z_s$ | 11<br>$\Delta T_s$ | 12<br>$T_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ------ | 30.09 | ------ | 80 | 0.0739 | ------ | ------ | 1,130 | ------ | [1] 1.40 | 1.40 |
| 2.5 | 2.5 | 28.05 | −2.04 | 72 | .0700 | 0.0720 | 2,020 | 2,940 | 1,810 | 2.24 | 3.64 |
| 5.0 | 2.5 | 26.67 | −1.38 | 66 | .0673 | .0686 | 1,423 | 4,250 | 1,310 | 2.30 | 5.94 |
| 7.5 | 2.5 | 24.97 | −1.70 | 60 | .0637 | .0655 | 1,837 | 6,100 | 1,850 | 2.52 | 8.46 |
| 10.0 | 2.5 | 23.50 | −1.47 | 56 | .0604 | .0621 | 1,675 | 7,800 | 1,700 | 2.54 | 11.00 |
| 12.5 | 2.5 | 22.50 | −1.00 | 50 | .0585 | .0594 | 1,190 | 8,880 | 1,080 | 2.27 | 13.27 |
| 15.0 | 2.5 | 21.25 | −1.25 | 46 | .0557 | .0581 | 1,588 | 10,500 | 1,620 | 2.55 | 15.82 |
| 17.5 | 2.5 | 20.10 | −1.15 | 45 | .0528 | .0542 | 1,501 | 12,100 | 1,600 | 2.66 | 18.48 |
| 20.0 | 2.5 | 19.00 | −1.10 | 43 | .0501 | .0515 | 1,510 | 13,800 | 1,700 | 2.71 | 21.19 |
| 22.5 | 2.5 | 18.05 | −.95 | 39 | .0479 | .0490 | 1,371 | 15,200 | 1,400 | 2.55 | 23.74 |
| 25.0 | 2.5 | 17.25 | −.80 | 34 | .0463 | .0471 | 1,202 | 16,250 | 1,050 | 2.18 | 25.92 |
| 27.5 | 2.5 | 16.72 | −.53 | 30 | .0452 | .0457 | 820 | 17,000 | 750 | 2.28 | 28.20 |
| 30.0 | 2.5 | 15.95 | −.77 | 30 | .0432 | .0442 | 1,233 | 18,350 | 1,350 | 2.73 | 30.93 |
| 32.5 | 2.5 | 15.60 | −.35 | 26 | .0425 | .0428 | 578 | 18,750 | 400 | 1.73 | 32.66 |
| 35.0 | 2.5 | 15.15 | −.45 | 24 | .0415 | .0420 | 757 | 19,500 | 750 | 2.48 | 35.14 |
| 37.5 | 2.5 | 14.80 | −.35 | 22 | .0407 | .0411 | 602 | 20,100 | 600 | 2.50 | 37.64 |
| 40.0 | 2.5 | 14.06 | −.74 | 19 | .0389 | .0398 | 1,315 | 21,350 | 1,250 | 2.38 | 40.02 |
| 42.5 | 2.5 | 13.75 | −.31 | 15 | .0383 | .0386 | 568 | 21,800 | 450 | 1.98 | 42.00 |
| 45.0 | 2.5 | 13.26 | −.49 | 11 | .0373 | .0378 | 916 | 22,550 | 750 | 2.04 | 44.04 |
| 47.5 | 2.5 | 12.90 | −.36 | 9 | .0364 | .0368 | 692 | 23,300 | 750 | 2.71 | 46.75 |
| 50.0 | 2.5 | 12.50 | −.40 | 5 | .0357 | .0360 | 786 | 23,900 | 600 | 1.91 | 48.66 |
| 52.5 | 2.5 | 12.10 | −.40 | 0 | .0348 | .0353 | 802 | 24,500 | 600 | 1.83 | 50.49 |
| 55.0 | 2.5 | 11.87 | −.23 | −1 | .0342 | .0345 | 471 | 25,100 | 600 | 3.18 | 53.67 |
| 57.5 | 2.5 | 11.67 | −.20 | −2 | .0337 | .0339 | 418 | 25,500 | 400 | 2.39 | 56.06 |
| 60.0 | 2.5 | 11.50 | −.17 | −3 | .0333 | .0335 | 359 | 25,800 | 300 | 2.09 | 58.15 |
| 62.5 | 2.5 | 11.30 | −.20 | −5 | .0330 | .0331 | 427 | 26,100 | 300 | 1.76 | 59.91 |
| 65.0 | 2.5 | 11.03 | −.27 | −7 | .0323 | .0326 | 586 | 26,650 | 550 | 2.34 | 62.25 |
| 67.5 | 2.5 | 10.88 | −.15 | −9 | .0319 | .0321 | 332 | 27,000 | 350 | 2.64 | 64.88 |
| 68.0 | 0.5 | 10.80 | −.08 | −10 | .0318 | .0318 | 178 | 27,100 | 100 | 0.28 | 65.16 |

[1] 1.40 is an initial time increment equal to the time required to climb from zero altitude in the standard atmosphere to the starting point, which is at 1,130 feet.

## PART III

# THE CONVERSION OF AIRSPEED INDICATOR READINGS INTO TRUE AIR SPEEDS

## THE AIRSPEED INDICATOR

By Walter S. Diehl

### SUMMARY

This part describes the conventional method of calibrating airspeed indicators and reducing indicated airspeeds to true airspeeds in the standard atmosphere.

### INTRODUCTION

In order that the method of converting airspeed indicator readings into true airspeeds may be fully understood, a brief explanation of the functioning of the instrument will be given.

An airspeed indicator consists essentially of two parts, the head and the indicator. The head, when placed in a relatively moving fluid, provides a differential pressure $p$, which is directly proportional to the density of the fluid $\rho$ and to the square of relative velocity $V$. That is,

$$p = p_0 - p_1 = K\rho V^2 \qquad (1)$$

For any given type and size of head, $K$ is constant over the usual range of flight velocities and its value may readily be determined by simple tests. When $K$ is known, the value of the differential pressure $p$ may be calculated for any desired values of $V$ and $\rho$. The indicators, or gauges, are designed to operate on the differential pressure produced by the head. The scale is graduated to read directly the relative velocity corresponding to each differential pressure when $\rho$ has the fixed value 0.00237 (lb.–ft.–sec. units), known as the standard density. If the instrument be properly constructed and graduated, the relative velocities or airspeeds will be correctly indicated in air of standard density. When the air density differs from the standard value the readings will be indicated airspeeds, $V_i$, and not true airspeeds, $V$. More properly, the reading of the instrument is always an indicated airspeed, which is a true airspeed only when the air density has the standard value used in the design of the gauge. The relation between $V_i$ and $V$ is obviously

$$V_i = \sqrt{\rho/\rho_0}\,V \quad \text{-----} \quad 2$$

since $p/K$ will be constant at any given reading.

Owing to the effects of interference from the parts of the airplane surrounding the head, and to variations in the construction of instruments and in the methods of installation, an airspeed indicator very rarely registers the true indicated airspeed. In order to reduce an airspeed indicator reading $V_a$ to the true indicated airspeed, $V_i$, it is therefore necessary to calibrate the installation over the entire range of speeds employed and thus obtain a curve of true indicated airspeed against airspeed indicator reading. Note that we are using three speeds, (1) the airspeed indicator reading, $V_a$, (2) the true indicated airspeed, $V_i$, and (3) the true airspeed, $V$. The ground speed does not enter into readings of airspeed except in calibration tests.

The curve of $V_i$ against $V_a$ is known as the calibration curve, and once established for any particular installation it may be used to convert any normal airspeed indicator reading into the corresponding true airspeed, as will be shown later. The conventional method of deriving the calibration curve will first be explained.

## AIRSPEED INDICATOR CALIBRATION

The calibration of an airspeed indicator is simply a determination of the true indicated airspeeds, $V_i$, and corresponding airspeed indicator readings, $V_a$, at a sufficient number of points to determine the curve of $V_i$ against $V_a$. Since $V_i$ is the speed that should be indicated and $V_a$ is the speed actually indicated, any method which will determine the value of $V_i$ corresponding to $V_a$ will be satisfactory. The usual method is to make timed runs with and against the wind over a measured course. In this method the wind must be approximately parallel to the course, or of comparatively low velocity. The flights should be made at a low and uniform altitude, the pilot using a statoscope if necessary. The timing must be accurate, and only high-grade, properly regulated stop watches or chronographs should be used. Particular care must be taken if the timing is done by the pilot or an observer to see that the starting and stopping of the watch are made under the same conditions. Obviously the length of the course must be accurately known and the ends well defined.

Five sets of runs are usually sufficient, a set being understood to consist of two runs at constant airspeed indicator reading, one with and one against the wind. These runs should be so made that one set is at high speed, one set at the lowest safe flying speed, and the three remaining sets at approximately equal intermediate intervals.

In all calibration tests the following data must be taken: (*a*) Length of course, barometric pressure, air temperature; (*b*) altimeter reading, average airspeed indicator reading for each run, time over the course for each run, and air temperature. Note that the readings in group (*a*) are as necessary as the actual flight data in group (*b*). These data are reduced to a calibration curve, first by calculating the ground speed over the course with the wind and against the wind, and taking the average of the two as the true airspeed at the given meter reading. Note

that this true airspeed must be obtained from the average of the two speeds and not from the average time over the course. The next step is to calculate the true indicated airspeed, $V_i$; that is, the airspeed which would have been indicated had the instrument read correctly, according to equation 2.

$$V_i = V\sqrt{\rho/\rho_o} \quad \text{-----} \quad 2$$

The values of $\sqrt{\rho/\rho_o}$ may be read directly from Figure 5 when the barometric pressure and air temperature are known. The barometric pressure should be corrected for the flight altitude. The observed air temperature is used.

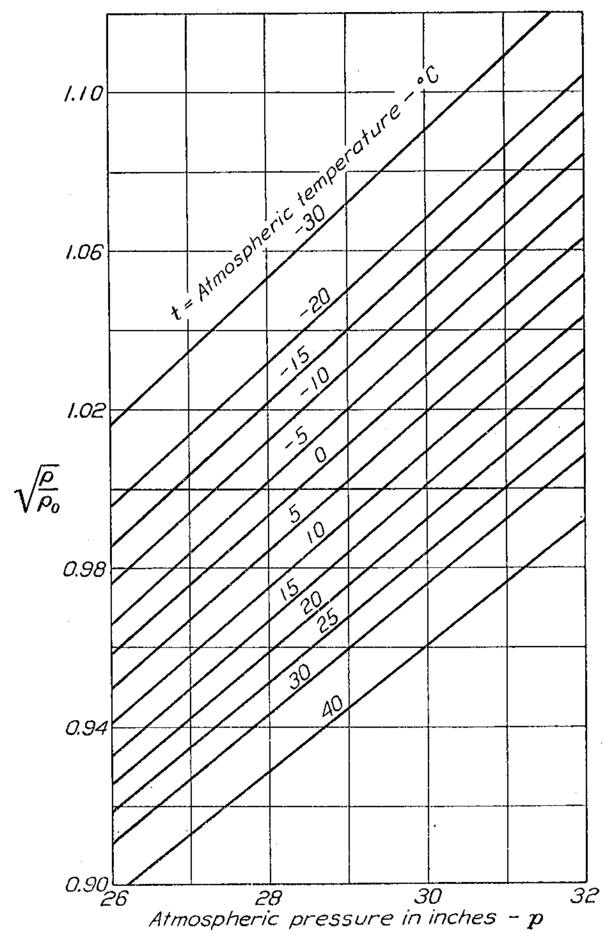



FIG. 5.—Variation of $\sqrt{\frac{\rho}{\rho_o}}$ with $p$ and $t$ for use in calibrating airspeed meters

The plot of $V_i$ against $V_a$ is usually a straight line. Even in extreme cases the curvature is so slight that the calibration curve may be extended considerably beyond the observed limits without introducing appreciable error, except at very low speeds.

Table V illustrates a convenient method of tabulating the calculations necessary to derive a calibration curve from test data.

## REDUCTION OF AIRSPEED INDICATOR READINGS TO TRUE AIRSPEEDS

The reduction of airspeed indicator readings $V_a$ to true airspeeds consists of two steps. The true indicated airspeed, $V_i$, corresponding to $V_a$, is first read from the calibration curve. $V_i$ is then converted to the true airspeed by the use of equation 2. That is,

$$V = V_i\sqrt{\frac{\rho_o}{\rho}} \quad \text{-----} \quad 2a$$

A convenient method of tabulating the calculations is given in Table V. The following steps are taken in the order given: The test data are put down in three columns, (1) altimeter reading, $Z_i$, (2) average airspeed meter reading, $V_a$, and (3) air temperature, $t$. The value of $\left(\frac{p}{p_o}\right)$ corresponding to $Z_i$ is then read from Figure 1 and placed in column 4. $\frac{p}{p_o}$ determines $p$, since $p_o$ is known. The equivalent altitude in standard atmosphere, $Z_s$, corresponding to $p$ and $t$, is read from Figure 2. The value of $\sqrt{\frac{\rho_o}{\rho}}$ is read from Figure 6 and the value of $V_i$ corresponding to $V_a$ is read from the calibration curve. $V$, the true airspeed, is the product of $V_i$ and $V$, and is plotted against $Z_e$ to obtain the variation of velocity with altitude.

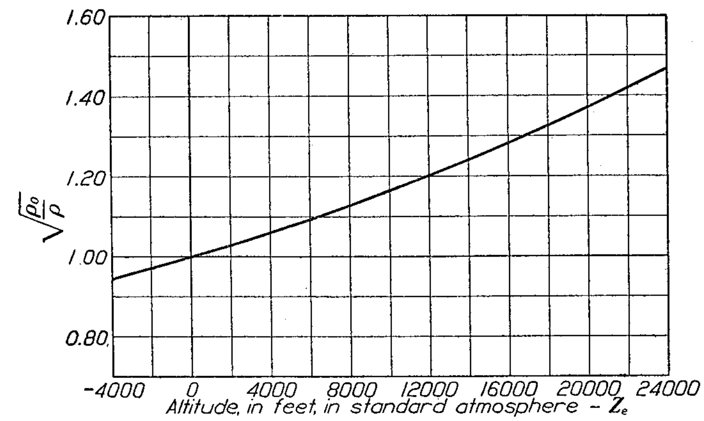



FIG. 6.—Variation of $\sqrt{\frac{\rho_o}{\rho}}$ with $Z$ in standard atmosphere

## TABLE V

### AIRSPEED METER CALIBRATION

Airplane type: ---------- No. ---------- Gross weight: ----------

Place: ---------- Date: ---------- Pilot: ----------

Weather: Wind velocity {Miles per hour ---------- / Knots ---------- Direction: ---------- Barometer: ---------- Temperature: ----------

Length of course $L$ {Land miles ---------- / Nautical miles ----------

Airspeed meter: Type ----------

Readings in {Miles per hour ---------- / Knots ----------

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Run | Altitude reading | Average airspeed meter reading | Time over course | | Speed over course | | Average air-speed | Air tempera-ture, °C. | Air pressure, $p$, inches | $\sqrt{\frac{\rho}{\rho_o}}$ | Cor-rected indicated airspeed, $V_i$, (11) by (8) | Remarks |
| | | | Against wind | With wind | Against wind | With wind | | | | | | |
| | $Z_i$ | $V_a$ | $T_1$ | $T_2$ | $V_1$ | $V_2$ | $V$ | | | | | |
| 1 | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |

NOTE.— $V_1=\frac{3600L}{T_1\text{ sec.}}$, $V_2\ \frac{3600L}{T_2\text{sec.}}$, $V=\frac{V_1+V_2}{2}$

Plot $V_a$ (col. 3) against $V_i$ (col. 12) for correction curve.

# APPENDIX

## TABLE VI

### STANDARD ATMOSPHERE TABLES AND CURVES

### STANDARD ATMOSPHERE METRIC UNITS

| Altitude, $Z$, meters | $t$ °C | $\frac{T}{T_o}$ | $\frac{p}{p_o}$ | $\frac{\rho}{\rho_o}$ | Pressure, $p$, mm. | Specific weight, $g\rho$, kg./m.³ |
|---|---|---|---|---|---|---|
| −1,000 | 21.50 | 1.0226 | 1.1244 | 1.0996 | 854.58 | 1.3476 |
| −800 | 20.20 | 1.0181 | 1.0986 | 1.0791 | 834.94 | 1.3325 |
| −600 | 18.90 | 1.0135 | 1.0733 | 1.0589 | 815.67 | 1.2977 |
| −400 | 17.60 | 1.0091 | 1.0484 | 1.0390 | 796.75 | 1.2733 |
| −200 | 16.30 | 1.0045 | 1.0240 | 1.0193 | 778.20 | 1.2492 |
| 0 | 15.00 | 1.0000 | 1.0000 | 1.0000 | 760.00 | 1.2255 |
| +200 | 13.70 | .9955 | .9765 | .9809 | 742.12 | 1.2021 |
| 400 | 12.40 | .9910 | .9534 | .9621 | 724.62 | 1.1791 |
| 600 | 11.10 | .9865 | .9308 | .9436 | 704.45 | 1.1564 |
| 800 | 9.80 | .9820 | .9087 | .9254 | 690.60 | 1.1340 |
| 1,000 | 8.50 | .9775 | .8870 | .9074 | 674.09 | 1.1120 |
| 1,200 | 7.20 | .9729 | .8656 | .8897 | 657.89 | 1.0903 |
| 1,400 | 5.90 | .9684 | .8448 | .8723 | 642.00 | 1,0690 |
| 1,600 | 4.60 | .9639 | .8243 | .8551 | 626.44 | 1.0480 |
| 1,800 | 3.30 | .9594 | .8042 | .8382 | 611.19 | 1.0272 |
| 2,000 | 2.00 | .9549 | .7845 | .8216 | 596.23 | 1,0068 |
| 2,200 | 0.70 | .9504 | .7652 | .8052 | 581.56 | .9868 |
| 2,400 | −0.60 | .9459 | .7463 | .7891 | 567.19 | .9670 |
| 2,600 | −1.90 | .9413 | .7278 | .7732 | 553.10 | .9475 |
| 2,800 | −3.20 | .9368 | .7097 | .7575 | 539.32 | .9283 |
| 3,000 | −4.50 | .9323 | .6918 | .7420 | 525.79 | .9094 |
| 3,200 | −5.80 | .9278 | .6744 | .7269 | 512.56 | .8908 |
| 3,400 | −7.10 | .9233 | .6574 | .7120 | 499.58 | .8726 |
| 3,600 | −8.40 | .9188 | .6406 | .6972 | 486.88 | .8545 |
| 3,800 | −9.70 | .9143 | .6242 | .6828 | 474.44 | .8368 |
| 4,000 | −11.00 | .9097 | .6082 | .6686 | 462.26 | .8193 |
| 4,200 | −12.30 | .9052 | .5925 | .6545 | 450.32 | .8022 |
| 4,400 | −13.60 | .9007 | .5771 | .6408 | 438.64 | .7853 |
| 4,600 | −14.90 | .8962 | .5621 | .6273 | 427.22 | .7687 |
| 4,800 | −16.20 | .8917 | .5474 | .6139 | 416.02 | .7523 |
| 5,000 | −17.50 | .8872 | .5330 | .6008 | 405.09 | .7363 |
| 5,200 | −18.80 | .8827 | .5187 | .5879 | 394.36 | .7205 |
| 5,400 | −20.10 | .8781 | .5051 | .5752 | 383.88 | .7049 |
| 5,600 | −21.40 | .8736 | .4916 | .5627 | 373.61 | .6897 |
| 5,800 | −22.70 | .8692 | .4784 | .5505 | 363.59 | .6746 |
| 6,000 | −24.00 | .8646 | .4655 | .5384 | 353.77 | .6598 |
| 6,200 | −25.30 | .8601 | .4528 | .5265 | 344.17 | .6453 |
| 6,400 | −26.60 | .8556 | .4405 | .5149 | 334.80 | .6310 |
| 6,600 | −27.90 | .8511 | .4284 | .5034 | 325.62 | .6169 |
| 6,800 | −29.20 | .8466 | .4166 | .4921 | 316.65 | .6031 |
| 7,000 | −30.50 | .8420 | .4051 | .4810 | 307.87 | .5896 |
| 7,200 | −31.80 | .8375 | .3937 | .4702 | 299.29 | .5762 |
| 7,400 | −33.10 | .8330 | .3828 | .4595 | 290.90 | .5632 |
| 7,600 | −34.40 | .8285 | .3720 | .4490 | 282.72 | .5503 |
| 7,800 | −35.70 | .8240 | .3614 | .4384 | 274.71 | .5377 |
| 8,000 | −37.00 | .8195 | .3512 | .4285 | 266.89 | .5252 |
| 8,200 | −38.30 | .8149 | .3411 | .4185 | 259.26 | .5130 |
| 8,400 | −39.60 | .8104 | .3312 | .4088 | 251.79 | .5010 |
| 8,600 | −40.90 | .8059 | .3217 | .3992 | 244.52 | .4893 |
| 8,800 | −42.20 | .8014 | .3123 | .3898 | 237.40 | .4777 |
| 9,000 | −43.50 | .7969 | .3032 | .3806 | 230.45 | .4664 |
| 9,200 | −44.80 | .7924 | .2942 | .3715 | 223.68 | .4552 |
| 9,400 | −46.10 | .7879 | .2856 | .3625 | 217.06 | .4443 |
| 9,600 | −47.40 | .7833 | .2771 | .3538 | 210.62 | .4336 |
| 9,800 | −48.70 | .7788 | .2688 | .3452 | 204.30 | .4230 |
| 10,000 | −50.00 | .7743 | .2606 | .3367 | 198.16 | .4127 |
| 10,200 | −51.30 | .7698 | .2528 | .3285 | 192.16 | .4026 |
| 10,400 | −52.60 | .7653 | .2451 | .3204 | 186.31 | .3926 |
| 10,600 | −53.90 | .7608 | .2377 | .3124 | 180.61 | .3828 |
| 10,800 | −55.00 | .7569 | .2303 | .3044 | 175.06 | .3729 |
| 11,000 | −55.00 | .7569 | .2232 | .2950 | 169.66 | .3614 |
| 11,200 | −55.00 | .7569 | .2164 | .2859 | 164.43 | .3502 |
| 11,400 | −55.00 | .7569 | .2096 | .2770 | 159.34 | .3394 |
| 11,600 | −55.00 | .7569 | .2032 | .2684 | 154.43 | .3290 |
| 11,800 | −55.00 | .7569 | .1970 | .2601 | 149.67 | .3188 |
| 12,000 | −55.00 | .7569 | .1909 | .2521 | 145.05 | .3090 |
| 12,200 | −55.00 | .7569 | .1849 | .2443 | 140.57 | .2995 |
| 12,400 | −55.00 | .7569 | .1792 | .2368 | 136.24 | .2902 |
| 12,600 | −55.00 | .7569 | .1737 | .2295 | 132.04 | .2813 |
| 12,800 | −55.00 | .7569 | .1683 | .2224 | 127.96 | .2726 |
| 13,000 | −55.00 | .7569 | .1632 | .2155 | 124.01 | .2642 |
| 13,200 | −55.00 | .7569 | .1581 | .2089 | 120.19 | .2560 |
| 13,400 | −55.00 | .7569 | .1532 | .2024 | 116.48 | .2481 |
| 13,600 | −55.00 | .7569 | .1485 | .1961 | 112.90 | .2404 |
| 13,800 | −55.00 | .7569 | .1440 | .1901 | 109.41 | .2331 |
| 14,000 | −55.00 | .7569 | .1395 | .1843 | 106.02 | .2259 |
| 14,200 | −55.00 | .7569 | .1352 | .1786 | 102.75 | .2188 |
| 14,400 | −55.00 | .7569 | .1311 | .1731 | 99.58 | .2121 |
| 14,600 | −55.00 | .7569 | .1270 | .1677 | 96.50 | .2056 |
| 14,800 | −55.00 | .7569 | .1230 | .1625 | 93.52 | .1993 |
| 15,000 | −55.00 | .7569 | .1193 | .1576 | 90.65 | .1931 |

## TABLE VII

### STANDARD ATMOSPHERE ENGLISH UNITS

| Altitude, $Z$, feet | $t$ °F | $t$ °C | $\frac{T}{T_o}$ | $\frac{p}{p_o}$ | $\frac{\rho}{\rho_o}$ | Pressure, $p$, inches | Specific weight, $g\rho$, lb./ft.³ |
|---|---|---|---|---|---|---|---|
| −4,000 | 73.27 | 22.93 | 1.0275 | 1.1533 | 1.1225 | 34.51 | 0.08588 |
| −3,500 | 71.48 | 21.93 | 1.0241 | 1.1333 | 1.1066 | 33.91 | .08466 |
| −3,000 | 69.70 | 20.94 | 1.0206 | 1.1134 | 1.0909 | 33.31 | .08346 |
| −2,500 | 67.92 | 19.95 | 1.0172 | 1.0938 | 1.0753 | 32.72 | .08227 |
| −2,000 | 66.13 | 18.96 | 1.0138 | 1.0745 | 1.0599 | 32.15 | .08109 |
| −1,500 | 64.35 | 17.97 | 1.0103 | 1.0552 | 1.0446 | 31.58 | .07993 |
| −1,000 | 62.57 | 16.98 | 1.0069 | 1.0367 | 1.0296 | 31.02 | .07878 |
| −500 | 60.78 | 15.99 | 1.0034 | 1.0182 | 1.0147 | 30.47 | .07764 |
| 0 | 59.00 | 15.00 | 1.0000 | 1.0000 | 1.0000 | 29.92 | .07651 |
| +500 | 57.22 | 14.01 | .9966 | .9821 | .9855 | 29.38 | .07540 |
| 1,000 | 55.43 | 13.02 | .9931 | .9644 | .9710 | 28.86 | .07430 |
| 1,500 | 53.65 | 12.03 | .9897 | .9469 | .9568 | 28.33 | .07321 |
| 2,000 | 51.87 | 11.04 | .9862 | .9298 | .9428 | 27.82 | .07213 |
| 2,500 | 50.09 | 10.05 | .9828 | .9129 | .9288 | 27.31 | .07107 |
| 3,000 | 48.30 | 9.06 | .9794 | .8962 | .9151 | 26.81 | .07001 |
| 3,500 | 46,52 | 8.07 | .9759 | .8798 | .9015 | 26.32 | .06897 |
| 4,000 | 44.74 | 7.08 | .9725 | .8636 | .8881 | 25.84 | .06794 |
| 4,500 | 42.95 | 6.09 | .9690 | .8477 | .8748 | 25.36 | .06693 |
| 5,000 | 41.17 | 5.09 | .9656 | .8320 | .8616 | 24.89 | .06592 |
| 5,500 | 39.39 | 4.10 | .9622 | .8165 | .8487 | 24.43 | .06493 |
| 6,000 | 37.60 | 3.11 | .9587 | .8013 | .8358 | 23.98 | .06395 |
| 6,500 | 35.82 | 2.12 | .9553 | .7863 | .8232 | 23.53 | .06298 |
| 7,000 | 34.04 | 1.13 | .9518 | .7716 | .8016 | 23.09 | .06202 |
| 7,500 | 32.25 | 0.14 | .9484 | .7571 | .7982 | 22.65 | .06107 |
| 8,000 | 30.47 | −0.85 | .9450 | .7427 | .7859 | 22.22 | .06013 |
| 8,500 | 28.69 | −1.84 | .9415 | .7286 | .7738 | 21.80 | .05920 |
| 9,000 | 26.90 | −2.83 | .9381 | .7147 | .7619 | 21.39 | .05829 |
| 9,500 | 25.12 | −3.82 | .9346 | .7011 | .7501 | 20.98 | .05739 |
| 10,000 | 23.34 | −4.81 | .9312 | .6876 | .7384 | 20.58 | .05649 |
| 10,500 | 21.56 | −5.80 | .9278 | .6743 | .7269 | 20.18 | .05561 |
| 11,000 | 19.77 | −6.79 | .9243 | .6614 | .7154 | 19.79 | .05474 |
| 11,500 | 17.99 | −7.78 | .9209 | .6486 | .7042 | 19.41 | .05388 |
| 12,000 | 16.21 | −8.77 | .9175 | .6359 | .6931 | 19.03 | .05303 |
| 12,500 | 14.42 | −9.77 | .9140 | .6234 | .6821 | 18.65 | .05219 |
| 13,000 | 12.64 | −10.76 | .9106 | .6112 | .6712 | 18.28 | .05136 |
| 13,500 | 10.86 | −11.75 | .9071 | .5992 | .6605 | 17.93 | .05054 |
| 14,000 | 9.07 | −12.74 | .9037 | .5873 | .6499 | 17.57 | .04973 |
| 14,500 | 7.29 | −13.73 | .9003 | .5757 | .6394 | 17.22 | .04893 |
| 15,000 | 5.51 | −14.72 | .8968 | .5642 | .6291 | 16.88 | .04814 |
| 15,500 | 3.72 | −15.71 | .8934 | .5530 | .6189 | 16.54 | .04736 |
| 16,000 | 1.94 | −16.70 | .8899 | .5418 | .6088 | 16.21 | .04658 |
| 16,500 | 0.16 | −17.69 | .8865 | .5309 | .5988 | 15.89 | .04583 |
| 17,000 | −1.63 | −18.68 | .8831 | .5202 | .5891 | 15.57 | .04507 |
| 17,500 | −3.41 | −19.67 | .8796 | .5097 | .5793 | 15.25 | .04433 |
| 18,000 | −5.19 | −20.66 | .8762 | .4992 | .5698 | 14.94 | .04359 |
| 18,500 | −6.97 | −21.65 | .8727 | .4891 | .5603 | 14.63 | .04287 |
| 19,000 | −8.76 | −22.64 | .8693 | .4790 | .5509 | 14.33 | .04216 |
| 19,500 | −10.54 | −23.63 | .8659 | .4691 | .5418 | 14.03 | .04145 |
| 20,000 | −12.32 | −24.62 | .8624 | .4594 | .5327 | 13.74 | .04075 |
| 20,500 | −14.10 | −25.62 | .8590 | .4498 | .5237 | 13.47 | .04007 |
| 21,000 | −15.89 | −26.61 | .8555 | .4405 | .5148 | 13.18 | .03938 |
| 21,500 | −17.67 | −27.60 | .8521 | .4313 | .5061 | 13.91 | .03872 |
| 22,000 | −19.46 | −28.59 | .8487 | .4222 | .4974 | 12.63 | .03806 |
| 22,500 | −21.24 | −29.58 | .8452 | .4133 | .4899 | 12.36 | .03740 |
| 23,000 | −23.02 | −30.57 | .8418 | .4045 | .4805 | 12.10 | .03676 |
| 23,500 | −24.81 | −31.56 | .8383 | .3959 | .4721 | 11.84 | .03612 |
| 24,000 | −26.59 | −32.55 | .8349 | .3874 | .4640 | 11.59 | .03550 |
| 24,500 | −28.371 | −33.54 | .8315 | .3791 | .4559 | 11.34 | .03488 |
| 25,000 | −30.15 | −34.53 | .8280 | .3709 | .4480 | 11.10 | .03427 |
| 25,500 | −31.94 | −35.52 | .8246 | .3629 | .4401 | 10.86 | .03367 |
| 26,000 | −33.72 | −36.51 | .8211 | .3550 | .4323 | 10.62 | .03308 |
| 26,500 | −35.50 | −37.50 | .8177 | .3473 | .4247 | 10.39 | .03249 |
| 27,000 | −37.29 | −38.49 | .8143 | .3397 | .4171 | 10.16 | .03192 |
| 27,500 | −39.07 | −39.48 | .8108 | .3322 | .4097 | 9.94 | .03134 |
| 28,000 | −40.85 | −40.47 | .8074 | .3248 | .4023 | 9.72 | .03078 |
| 28,500 | −42.64 | −41.46 | .8039 | .3176 | .3951 | 9.50 | .03023 |
| 29,000 | −44.42 | −42.46 | .8005 | .3106 | .3869 | 9.29 | .02968 |
| 29,500 | −46.20 | −43.45 | .7971 | .3035 | .3820 | 9.09 | .02914 |
| 30,000 | −47.99 | −44.44 | .7936 | .2968 | .3740 | 8.88 | .02861 |
| 30,500 | −49.77 | −45.43 | .7902 | .2900 | .3671 | 8.68 | .02809 |
| 31,000 | −51.55 | −46.42 | .7867 | .2834 | .3603 | 8.48 | .02757 |
| 31,500 | −53.33 | −47.41 | .7833 | .2770 | .3537 | 8.29 | .02706 |
| 32,000 | −55.12 | −48.40 | .7799 | .2707 | .3472 | 8.10 | .02656 |
| 32,500 | −56.90 | −49.39 | .7764 | .2645 | .3407 | 7.91 | .02606 |
| 33,000 | −58.68 | −50.38 | .7730 | .2583 | .3343 | 7.73 | .02558 |
| 33,500 | −60.47 | −51.39 | .7695 | .2524 | .3280 | 7.55 | .02510 |
| 34,000 | −62.25 | −52.36 | .7661 | .2465 | .3218 | 7.38 | .02463 |
| 34,500 | −64.03 | −53.35 | .7627 | .2408 | .3157 | 7.21 | .02416 |
| 35,000 | −65.82 | −54.34 | .7592 | .2352 | .3098 | 7.04 | .02369 |
| 35,500 | −67.00 | −55.00 | .7569 | .2296 | .3033 | 6.87 | .02321 |
| 36,000 | −67.00 | −55.00 | .7569 | .2242 | .2962 | 6.71 | .02265 |
| 37,000 | −67.00 | −55.00 | .7569 | .2137 | .2824 | 6.39 | .02160 |
| 38,000 | −67.00 | −55.00 | .7569 | .2037 | .2692 | 6.10 | .02059 |
| 39,000 | −67.00 | −55.00 | .7569 | .1943 | .2566 | 5.81 | .01963 |
| 40,000 | −67.00 | −55.00 | .7569 | .1852 | .2447 | 5.54 | .01872 |
| 41,000 | −67.00 | −55.00 | .7569 | .1765 | .2332 | 5.28 | .01785 |
| 42,000 | −67.00 | −55.00 | .7569 | .1683 | .2224 | 5.04 | .01701 |
| 43,000 | −67.00 | −55.00 | .7569 | .1605 | .2120 | 4.80 | .01622 |
| 44,000 | −67.00 | −55.00 | .7569 | .1530 | .2021 | 4.58 | .01546 |
| 45,000 | −67.00 | −55.00 | .7569 | .1458 | .1926 | 4.37 | .01474 |
| 46,000 | −67.00 | −55.00 | .7569 | .1391 | .1837 | 4.16 | .01405 |
| 47,000 | −67.00 | −55.00 | .7569 | .1325 | .1751 | 3.97 | .01339 |
| 48,000 | −67.00 | −55.00 | .7569 | .1264 | .1669 | 3.781 | .01277 |
| 49,000 | −67.00 | −55.00 | .7569 | .1205 | .1591 | 3.60 | .01217 |
| 50,000 | −67.00 | −55.00 | .7569 | .1149 | .1517 | 3.44 | .01161 |

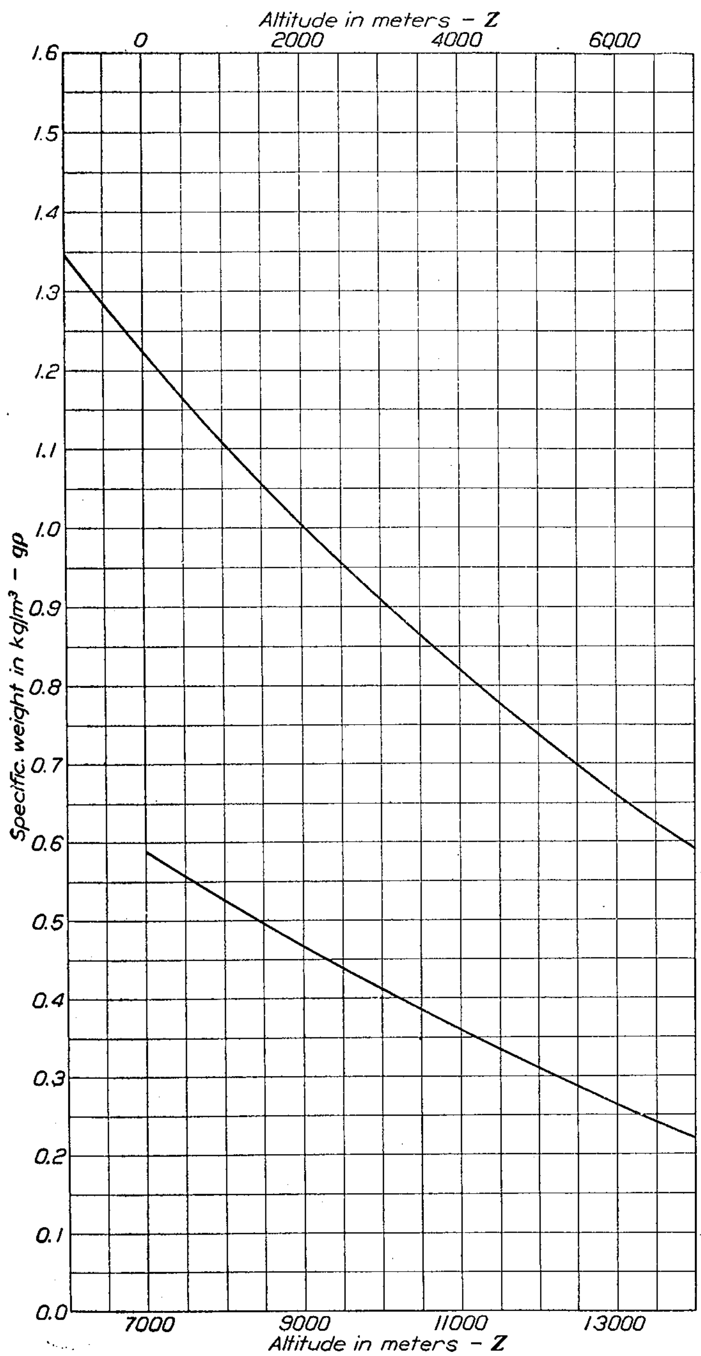



FIG. 7.—Altitude—Specific weight. Standard atmosphere. Metric units

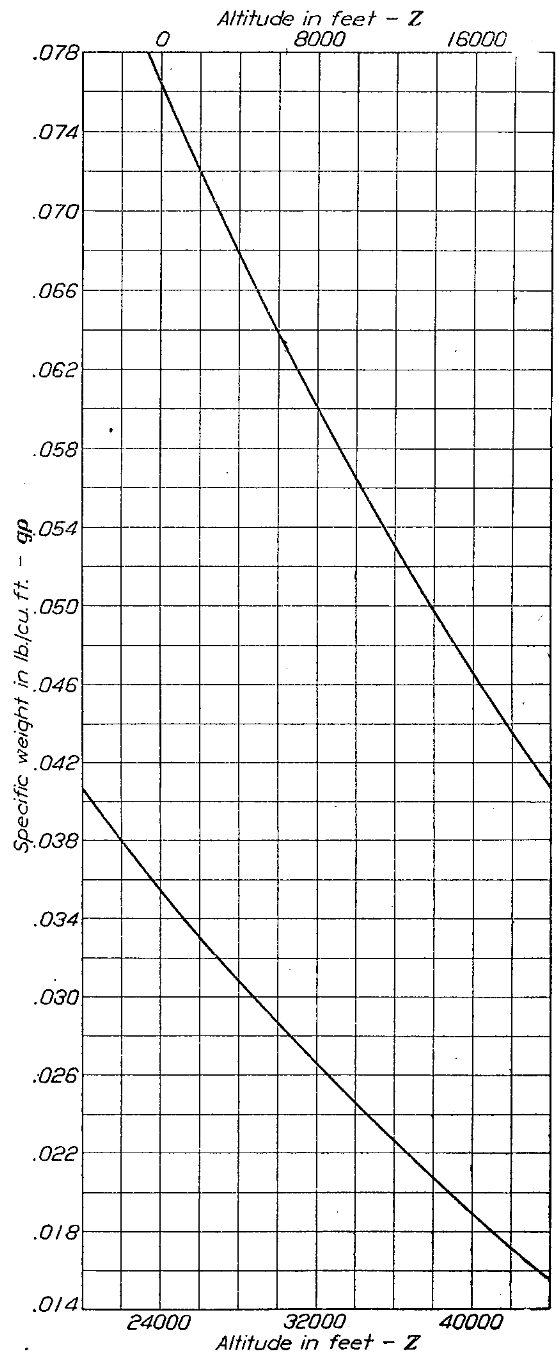
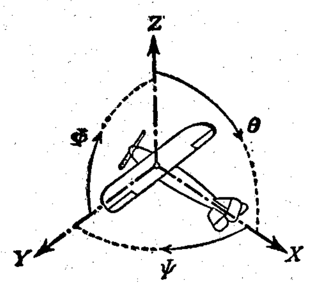



FIG. 8.—Altitude—Specific weight. Standard atmosphere. English units




Positive directions of axes and angles (forces and moments) are shown by arrows.

| Axis. | | Force (parallel to axis) symbol. | Moment about axis. | | | Angle. | | Velocities. | |
|---|---|---|---|---|---|---|---|---|---|
| Designation. | Sym-bol. | | Designa-tion. | Sym-bol. | Positive direc-tion. | Designa-tion. | Sym-bol. | Linear (compo-nent along axis). | Angular. |
| Longitudinal | $X$ | $X$ | rolling | $L$ | $Y \longrightarrow Z$ | roll | $\Phi$ | $u$ | $p$ |
| Lateral | $Y$ | $Y$ | pitching | $M$ | $Z \longrightarrow X$ | pitch | $\Theta$ | $v$ | $q$ |
| Normal | $Z$ | $Z$ | yawing | $N$ | $X \longrightarrow Y$ | yaw | $\Psi$ | $w$ | $r$ |

Absolute coefficients of moment

$$C_l = \frac{L}{q b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q f S}$$

Angle of set of control surface (relative to neutral position), $\delta$. (Indicate surface by proper subscript.)

**4. PROPELLER SYMBOLS.**

Diameter, $D$
Pitch (a) Aerodynamic pitch, $p_a$
(b) Effective pitch, $p_e$
(c) Mean geometric pitch, $p_g$
(d) Virtual pitch, $p_v$
(e) Standard pitch, $p_s$
Pitch ratio, $p/D$
Inflow velocity, $V'$
Slipstream velocity, $V_s$

Thrust, $T$
Torque, $Q$
Power, $P$
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T V/P$
Revolutions per sec., $n$; per min., $N$
Effective helix angle $\Phi = \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

**5. NUMERICAL RELATIONS.**

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.